Design of two-channel IIR linear phase PR filter banks

Xi Zhang*, Toshinori Yoshikawa

Department of Electrical Engineering, Nagaoka University of Technology, 1603-1 Kamitomioka-machi, Nagaoka, Niigata, 940-2188, Japan

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Abstract

In this paper, a novel method is proposed for designing two-channel biorthogonal perfect reconstruction filter banks with exact linear phase using noncausal IIR filters. Since the structurally perfect reconstruction implementation is adopted, the proposed filter banks are guaranteed to be a perfect reconstruction even when all filter coefficients are quantized. From the viewpoint of wavelets, design of biorthogonal IIR linear phase filter banks with an additional flatness constraint is considered. The proposed design method is based on the formulation of a generalized eigenvalue problem by using Remez exchange algorithm. Hence, the filter coefficients can be obtained by solving the eigenvalue problem to compute the positive minimum eigenvalue, and the optimal solution in the Chebyshev sense is easily obtained through a few iterations. The proposed procedure is computationally efficient, and the flatness constraint can be arbitrarily specified. Some design examples are presented to demonstrate the effectiveness of the proposed method.

Zusammenfassung


Résumé

Dans cet article, nous proposons une nouvelle méthode de conception de bancs de filtres à reconstruction parfaite, à phase linéaire exacte, biorthogonaux à deux canaux, en utilisant des filtres IIR non-causaux. Comme nous adoptons l’implémentation à reconstruction structurellement parfaite, il est garanti que les bancs de filtres proposés sont à reconstruction parfaite même quand tous les coefficients des filtres sont quantifiés. Du point de vue des ondelettes, nous

*Corresponding author. Tel.: +81 258 47 9522; fax: +81 258 47 9500; e-mail: xiz@nagaokaut.ac.jp.
considerons la conception de bancs de filtres IIR biorthogonaux à phase linéaire avec une contrainte supplémentaire de platitude. La méthode de conception proposée repose sur la formulation d’un problème aux valeurs propres généralisé, en utilisant un algorithme d’échange de Remez. Donc, les coefficients des filtres peuvent être obtenus par la résolution du problème des valeurs propres pour calculer la valeur propre positive minimale, et la solution optimale au sens de Chebyshev est obtenue aisément par quelques itérations. La procédure proposée est efficace en termes de temps de calcul, et la contrainte de platitude peut être spécifiée de façon arbitraire. Quelques exemples de conception sont présentés pour démontrer l’efficacité de la méthode proposée. © 1999 Published by Elsevier Science B.V. All rights reserved.

Keywords: IIR filter bank; Structurally perfect reconstruction; Linear phase; Eigenvalue problem

1. Introduction

Two-channel perfect reconstruction (PR) filter banks have been used in different applications of signal processing, such as subband coding of speech and image signals, transmultiplexers, and voice privacy systems [1–17]. The theory and design of FIR PR filter banks have been well established in recent years [2–8,10–17]. The PR filter banks include two cases: orthonormal and biorthogonal. For orthonormal case, the FIR PR filter banks, except Haar function, cannot possess exact linear phase responses that are desired in some applications of image signal processing. Thus, biorthogonal PR filter banks are proposed to obtain exact linear phase responses. Design of biorthogonal FIR linear phase PR filter banks has been also discussed in the previous works [2,3,5,7]. However, compared with IIR filters, FIR filters generally require higher-order filters for meeting the same magnitude specification. Hence, using IIR filters will result in computational savings [12]. In this paper, we will consider the design of biorthogonal PR filter banks with exact linear phase responses using IIR filters. Although causal IIR filters can possess only approximately linear phase responses, we can obtain an exact linear phase by using noncausal IIR filters, which can be decomposed into causal and anticausal parts to implement, and then anticausal part can be realized by using time reversal for finite length inputs. For example, in the subband image coding systems, the generalized circular convolution and symmetric extension methods can be used to treat the boundaries of images [12]. In most designs, the PR property of filter banks cannot be preserved generally when the filter coefficients are quantized. To force the PR condition to be satisfied regardless of what the filter coefficients are, the structurally PR implementation will be required. In [7], an efficient structurally PR implementation has been proposed, where for the FIR case, exact linear phase filters are used, and for the IIR case, causal allpass filters are used.

In this paper, we propose a new method for designing two-channel biorthogonal linear phase PR filter banks by using noncausal IIR filters. We adopt the structurally PR implementation proposed in [7], thus the obtained IIR filter banks still satisfy PR condition even when all filter coefficients are quantized. It is well known [1,6,7,9–11,17] that wavelet bases can be generated from PR filter banks. Then, synthesis of wavelet bases has been reduced to design PR filter banks. From the regularity condition of wavelets, a flatness constraint is required to impose on the PR filter banks. In this paper, from the view point of wavelets, we consider the design of IIR linear phase PR filter banks with an additional flatness constraint. By using Remez exchange algorithm, we formulate the design problem in the form of a generalized eigenvalue problem [18,19]. Thus, by solving the eigenvalue problem to compute the positive minimum eigenvalue, we can get a set of filter coefficients as the corresponding eigenvector. Therefore, the optimal filter coefficients with an equiripple response can be easily obtained through a few iterations. The proposed procedure is computationally efficient, and the flatness constraint can be arbitrarily specified.

This paper is organized as follows. Section 2 describes an efficient structurally PR implementation. Section 3 presents a design method of IIR linear phase PR filter banks based on eigenvalue problem by using Remez exchange algorithm. Section 4 shows two design examples to demonstrate
the effectiveness of the proposed method. Conclusions are given in Section 5.

2. Structurally PR filter banks

In two-channel filter banks shown in Fig. 1, assume that $H_0(z)$ and $H_1(z)$ are analysis filters, and $G_0(z)$ and $G_1(z)$ are synthesis filters. It is well known that the relationship of input $X(z)$ and output $Y(z)$ of the filter banks is

$$
Y(z) = \frac{1}{2}[H_0(z)G_0(z) + H_1(z)G_1(z)]X(z)
+ \frac{1}{2}[H_0(-z)G_0(z) + H_1(-z)G_1(z)]X(-z).
$$

(1)

Then the perfect reconstruction condition is

$$
H_0(z)G_0(z) + H_1(z)G_1(z) = z^{-2K-1},
$$

$$
H_0(-z)G_0(z) + H_1(-z)G_1(z) = 0,
$$

(2)

where $K$ is integer. By using the polyphase matrix description,

$$
\begin{bmatrix}
H_0(z) \\
H_1(z)
\end{bmatrix}
= \begin{bmatrix}
H_{00}(z^2) & H_{01}(z^2) \\
H_{10}(z^2) & H_{11}(z^2)
\end{bmatrix}
\begin{bmatrix}
1 \\
z^{-1}
\end{bmatrix}
= H(z^2)\begin{bmatrix}
1 \\
z^{-1}
\end{bmatrix},
$$

(3)

$$
\begin{bmatrix}
G_0(z) \\
G_1(z)
\end{bmatrix}^T
= \begin{bmatrix}
z^{-1} \\
1
\end{bmatrix}^T
\begin{bmatrix}
G_{00}(z^2) & G_{01}(z^2) \\
G_{10}(z^2) & G_{11}(z^2)
\end{bmatrix}
= \begin{bmatrix}
z^{-1} \\
1
\end{bmatrix}^T
G(z^2),
$$

(4)

the perfect reconstruction condition of Eq. (2) becomes

$$
G(z)H(z) = \frac{z^{-K}}{2}\begin{bmatrix}
1 & 0 \\
0 & 1
\end{bmatrix} = \frac{z^{-K}}{2}I,
$$

(5)

where $I$ is the identity matrix. It is well known that

$$
\begin{bmatrix}
z^{-N} & 0 \\
-A(z) & 1
\end{bmatrix}
\begin{bmatrix}
1 \\
z^{-N}
\end{bmatrix}
= z^{-N}I,
$$

(6)

$$
\begin{bmatrix}
1 & B(z) \\
z^{-M} & -B(z)
\end{bmatrix}
\begin{bmatrix}
z^{-M} \\
0
\end{bmatrix}
= z^{-M}I,
$$

(7)

where $N$ and $M$ are integers, and $A(z)$ and $B(z)$ are arbitrary transfer functions. If we constitute $H(z)$ and $G(z)$ as

$$
H(z) = \begin{bmatrix}
z^{-M} & -B(z) \\
0 & 1
\end{bmatrix}
\begin{bmatrix}
1 & 0 \\
\frac{1}{2}A(z) & \frac{1}{2}z^{-N}
\end{bmatrix},
$$

(8)

$$
G(z) = \begin{bmatrix}
\frac{1}{2}z^{-N} & 0 \\
-Iz^{-M} & 1
\end{bmatrix}
\begin{bmatrix}
1 & B(z) \\
0 & z^{-M}
\end{bmatrix},
$$

(9)

then the perfect reconstruction condition of Eq. (5) is satisfied regardless of what $A(z)$ and $B(z)$ are, i.e., the PR filter banks can be still obtained even when the filter coefficients of $A(z)$ and $B(z)$ are quantized. Note that $K = N + M$. The structurally PR implementation proposed in [7] is shown in Fig. 2. Hence, the design problem of the filter bank will become design of the analysis or synthesis filters. From Eqs. (8) and (9), the transfer functions of analysis and synthesis filters are

$$
H_0(z) = z^{-2M} - B(z^2)H_1(z),
$$

$$
H_1(z) = \frac{1}{2}(z^{-2N-1} + A(z^2)),
$$

(10)

and

$$
G_0(z) = \frac{1}{2}(z^{-2N-1} - A(z^2)) = -H_1(-z),
$$

$$
G_1(z) = z^{-2M} + B(z^2)G_0(z) = H_0(-z),
$$

(11)

respectively. In the following, we will consider the design of the analysis filters $H_0(z)$ and $H_1(z)$.

![Fig. 1. Two-channel filter bank.](Image 75x102 to 231x148)

![Fig. 2. Structurally perfect reconstruction implementation.](Image 287x102 to 498x146)
3. Design of IIR linear phase PR filter banks

In this section, we describe design of IIR linear phase PR filter banks based on eigenvalue problem by using Remez exchange algorithm. Here, we use general IIR filters \( A(z) \) and \( B(z) \), that is,

\[
A(z) = \sum_{i=0}^{L_1} a_i z^{-i}, \quad B(z) = \sum_{i=0}^{L_2} b_i z^{-i},
\]

where \( L_1, L_2, L_3, L_4 \) are integers, \( a_i, b_i, c_i, d_i \) are real coefficients, and \( b_0 = d_0 = 1 \).

3.1. Desired magnitude responses

From Eq. (10), we have

\[
H_1(z) = \frac{1}{2} z^{-2N-1} \left\{ 1 + z^{2N+1} A(z^2) \right\},
\]

\[
= \frac{1}{2} z^{-2N-1} \left\{ 1 + \hat{A}(z^2) \right\},
\]

where

\[
\hat{A}(z) = z^{N+1/2} A(z) = z^{N+1/2} \sum_{i=0}^{L_1} a_i z^{-i},
\]

(15)

To obtain the exact linear phase, the filter coefficients of \( A(z) \) must be symmetric, that is,

\[
a_i = a_{N-i}, \quad b_i = b_{N-i}, \quad i = 0, 1, \ldots, L_1,
\]

(16)

When \( L_2 \) is odd, \( A(z) \) will have a pole at \( z = 1 \), i.e., \( \sum_{i=0}^{L_2} \left( -1 \right)^i b_i = 0 \). Hence, \( L_2 \) must be even.

To force \( \hat{A}(z) \) to be zero phase, we have \( N + \frac{1}{2} L_1 + \frac{1}{2} L_2 = 0 \), and \( L_1 \) and \( L_2 \) must satisfy

\[
L_1 = 2I_1 + 1,
\]

\[
L_2 = 2I_2,
\]

\[
L_1 - L_2 = 2N + 1,
\]

(17)

where \( I_1 \) and \( I_2 \) are integers. Then, \( \hat{A}(z) \) becomes zero phase, and its frequency response is given by

\[
\hat{A}(e^{j\omega}) = \frac{1}{2 b_{I_1} + \sum_{i=0}^{L_2-1} b_i \cos(I_2 - i)\omega}.
\]

(18)

Therefore, \( H_1(z) \) has an exact linear phase, and its magnitude response is

\[
|H_1(e^{j\omega})| = \frac{1}{2} \left\{ 1 + \hat{A}(e^{j\omega}) \right\}.
\]

(19)

To force \( H_1(z) \) to be lowpass, the desired magnitude response of \( \hat{A}(z^2) \) is

\[
\hat{A}(e^{j2\omega}) = 1, \quad 0 \leq \omega \leq \omega_p,
\]

(20)

\[
\hat{A}(e^{j2\omega}) = -1, \quad \omega_p < \omega \leq \pi,
\]

where \( \omega_p \) and \( \omega_s \) are the passband and stopband edge frequencies, respectively, and \( \omega_p + \omega_s = \pi \). From Eq. (18), we can see that

\[
\hat{A}(e^{j2\pi - \omega}) = -\hat{A}(e^{j\omega}),
\]

thus the desired magnitude response is reduced to

\[
\hat{A}(e^{j\omega}) = 1, \quad 0 < \omega < 2\omega_p.
\]

(22)

From Eq. (10), we have

\[
H_0(z) = z^{-2M} \left\{ 1 - z^{-2N+1} H_1(z) B(z^2) \right\}
\]

\[
= z^{-2M} \left\{ 1 - z^{-2M} H_1(z) \hat{B}(z^2) \right\}
\]

\[
= z^{-2M} \left\{ 1 - \frac{1}{2} (1 + \hat{A}(z^2)) \hat{B}(z^2) \right\},
\]

(23)

where

\[
\hat{B}(z) = z^{M-N-1/2} B(z) = z^{M-N-1/2} \sum_{i=0}^{L_2} c_i z^{-i}.
\]

(24)

Similarly, to force \( \hat{B}(z) \) to be zero phase, the following conditions must be satisfied:

\[
c_i = c_{L_3-i}, \quad i = 0, 1, \ldots, L_3,
\]

(25)

\[
d_i = d_{L_4-i}, \quad i = 0, 1, \ldots, L_4
\]

and

\[
L_3 = 2I_3 + 1,
\]

\[
L_4 = 2I_4,
\]

\[
L_3 - L_4 = 2(M - N) - 1,
\]

(26)

where \( I_3 \) and \( I_4 \) are integers. Hence, the frequency response of \( \hat{B}(z) \) is given by

\[
\hat{B}(e^{j\omega}) = \frac{\sum_{i=0}^{I_3} c_i \cos(I_3 - i + \frac{1}{2})\omega}{\frac{1}{2} d_{I_4} + \sum_{i=0}^{L_4-1} d_i \cos(I_4 - i)\omega}.
\]

(27)
Then $H_0(z)$ has an exact linear phase and its magnitude response is

$$|H_0(e^{j\omega})| = 1 - |H_1(e^{j\omega})| \tilde{B}(e^{j2\omega}).$$

(28)

Since $|H_1(e^{j\omega})| = 0$ in $[\omega_p, \pi]$, it is clear from Eq. (28) that $|H_0(e^{j\omega})| = 1$, that is, the band $[\omega_p, \pi]$ is the passband of $H_0(z)$. In $[0, \omega_p]$, $|H_1(e^{j\omega})| = 1$, ideally, to force $|H_0(e^{j\omega})| = 0$, then the desired magnitude response of $B(z)$ must be

$$\tilde{B}_d(e^{j\omega}) = 1, \quad 0 \leq \omega \leq 2\omega_p.$$

(29)

Therefore, the design problem of the filter banks will become approximation of $\tilde{A}(z)$ and $B(z)$. Note that $A(z)$ and $B(z)$ are noncausal due to the symmetric conditions of Eqs. (16) and (25). $A(z)$ and $B(z)$ must be decomposed into causal and anticausal parts to implement, then the anticausal part can be realized by using time reversal for finite length inputs, such as image signals.

### 3.2. Design of maximally flat filters

We consider design of $H_1(z)$, i.e., $A(z)$. First, we define an error function $E_d(\omega)$ between the desired and actual magnitude responses of $A(z)$ as

$$E_d(\omega) = 1 - \tilde{A}(e^{j\omega}) = \frac{\frac{1}{2}b_{t_1} + \sum_{i=0}^{t_2-1} b_i \cos(i_2 - i/\omega) - \sum_{i=0}^{t_1} a_i \cos(i_1 - i + 1/2)\omega}{\frac{1}{2}b_{t_2} + \sum_{i=0}^{t_1-1} b_i \cos(i_2 - i/\omega)}.$$  

(30)

The design purpose is to find a set of filter coefficients $a_i$ and $b_i$ to minimize $E_d(\omega)$ in the band $[0,2\omega_p]$. It is well known [1,6,7,9-11,17] that wavelet bases can be generated from PR filter banks, then the synthesis of wavelet bases are reduced to design PR filter banks. From the regularity condition of wavelets, the PR filter banks are required to satisfy certain flatness constraints, that is,

$$\frac{\partial^k|H_1(e^{j\omega})|}{\partial \omega^k} \bigg|_{\omega = \pi} = 0, \quad k = 0,1,\ldots,2J_1 + 1,$$

(31)

where $J_1$ is an integer and $0 \leq J_1 \leq I_1 + I_2$. When $J_1 = I_1 + I_2$, $H_1(z)$ will be a maximally flat filter and the corresponding wavelet has maximum-order regularity.

From Eqs. (19), (21) and (30), the flatness constraints of Eq. (31) are equivalent to

$$\tilde{A}(e^{j\omega}) = 1,$$

(32)

$$\frac{\partial^k\tilde{A}(e^{j\omega})}{\partial \omega^k} \bigg|_{\omega = 0} = 0, \quad k = 1,2,\ldots,2J_1 + 1,$$

(33)

Substituting Eq. (30) into Eq. (33), we get

$$\frac{1}{2}b_{t_1} + \sum_{i=0}^{t_2-1} b_i - \sum_{i=0}^{t_1} a_i = 0,$$

$$\sum_{i=0}^{t_2-1} b_i (I_2 - i/\omega)^{2k} - \sum_{i=0}^{t_1} a_i \left( I_1 - i + 1/2 \right)^{2k} = 0,$$

(34)

$$k = 1,2,\ldots,J_1.$$  

When the maximally flat filters are needed, we can solve the $(I_1 + I_2 + 1)$ linear equations of Eq. (34) to obtain the filter coefficients, since $J_1 = I_1 + I_2$ and $b_0 = 1$. Therefore, the maximally flat filters can be easily obtained.

### 3.3. Design of filters with given flatness

It is well known that the maximally flat filters are poorly selective. However, frequency selectivity is also thought of as a useful property for many applications. It is known in [10] that frequency selectivity and regularity somewhat contradict each other. For this reason, we consider design of IIR filters that have the best-possible frequency selectivity for a given flatness constraint. Assume that the flatness constraints of Eq. (31) are required where $J_1 < I_1 + I_2$. We want to obtain an equiripple response by using the remaining degrees of freedom. First, we select $(I_1 + I_2 - J_1 + 1)$ extremal frequencies $\omega_i$ in $[0,2\omega_p]$ as follows:

$$2\omega_p = \omega_0 > \omega_1 > \cdots > \omega_{2I_1 + I_2 - J_1} > 0.$$  

(35)
By using Remez exchange algorithm, we then formulate $E_\omega(\omega)$ as
\[ E_\omega(\omega) = 1 - \tilde{A}(e^{j\omega}) = (-1)^I \delta, \tag{36} \]
where $\delta$ ($> 0$) is the magnitude error, and the denominator polynomial of $E_\omega(\omega)$ must satisfy
\[ \frac{1}{2} b_{I_1} + \sum_{i=0}^{I_1-1} b_i \cos(i I_2 - \omega) \neq 0, \quad \text{for all} \ \omega. \tag{37} \]
Substituting Eq. (30) into Eq. (36), we rewrite Eqs. (34) and (36) in the matrix form as
\[
P A = \delta Q A, \tag{38}
\]
where $A = [a_0, a_1, \ldots, a_i, b_0, b_1, \ldots, b_{I_2}]^T$, and the matrices $P$ and $Q$ are given by
\[
P = \begin{bmatrix}
-1 & \cdots & -1 \\
-(I_1 + \frac{1}{2})^2 & \cdots & -(\frac{1}{2})^2 \\
& \ddots & \\
& & -1 \\
-(I_1 + \frac{1}{2})^2 I_1 & \cdots & -(\frac{1}{2})^2 I_1 \\
-\cos(I_1 + \frac{1}{2}) w_0 & \cdots & -\cos(I_1 + I_2 - J_1) w_0 \\
& \ddots & \\
& & \cos I_2 \omega_0 \\
-\cos(I_1 + \frac{1}{2}) w_0 (I_1 + I_2 - J_1) & \cdots & -\cos(I_1 + I_2 - J_1) w_0 \\
\end{bmatrix}
\]
\[
Q = \begin{bmatrix}
0 & \cdots & 0 & 0 & \cdots & 0 & 0 \\
& \ddots & \vdots & \ddots & \ddots & \ddots & \ddots \\
0 & \cdots & 0 & \cos I_2 \omega_0 & \cdots & \cos \omega_0 & \frac{1}{2} \\
& \ddots & \ddots & \ddots & \ddots & \ddots & \ddots \\
0 & \cdots & 0 & (-1)^{I_1 + I_2 - J_1} \cos I_2 \omega_0 (I_1 + I_2 - J_1) & \cdots & (-1)^{I_1 + I_2 - J_1} \cos \omega_0 (I_1 + I_2 - J_1) & \frac{1}{2} (-1)^{I_1 + I_2 - J_1} \\
\end{bmatrix}
\tag{39}
\]
\[
\begin{bmatrix}
1 & \cdots & 1 & \frac{1}{2} \\
I_2^2 & \cdots & 1 & 0 \\
& \ddots & \ddots & \ddots \\
& & 1 & 0 \\
& & & \frac{1}{2} \\
\end{bmatrix}
\tag{40}
\]

It should be noted that Eq. (38) is a generalized eigenvalue problem, i.e., $\delta$ is an eigenvalue, and $A$ is a corresponding eigenvector [18]. It is known in [18] that to obtain a solution that satisfies Eq. (37), we only need to find the eigenvector corresponding to the positive minimum eigenvalue. In this design problem, we have found that the positive minimum eigenvalue is the absolute minimum one. Therefore, a set of filter coefficients can be easily obtained. In order to achieve an equiripple magnitude response, we make use of an iteration procedure to get the optimal filter coefficients. The design algorithm is shown as follows.

3.4. Design algorithm

**Procedure**

**Begin**
1. Read $L_1, L_2, J_1$ and $\omega_p$.
2. Select initial extremal frequencies $\Omega_i$ ($i = 0, \ldots, I_1 + I_2 - J_1$) equally spaced in the band $[0, 2\omega_p]$.
3. Set $\omega_i = \Omega_i$ ($i = 0, 1, \ldots, I_1 + I_2 - J_1$).
4. Compute $P, Q$ by using Eqs. (39) and (40), then find the positive minimum eigenvalue of Eq. (38) to obtain the filter coefficients $a_i$ and $b_i$ that satisfies Eq. (37).
5. Search the peak frequencies of $E_s(\omega)$ within the band $[0,2\omega_p]$, and store these frequencies into the corresponding $\Omega_i$.

Until Satisfy the following condition for the prescribed small constant $\varepsilon$:

$$\left\{ \begin{array}{l} \sum_{i=0}^{l_1+l_2-J_1} |\Omega_i - \omega| \leq \varepsilon \\ \end{array} \right.$$

End.

3.5. Design of $H_0(z)$

We consider design of $H_0(z)$, i.e., $B(z)$. $B(z)$ can be similarly designed by using the design algorithm proposed in Section 3.4. It is seen from Eq. (28) that the magnitude response of $H_0(z)$ is dependent on that of both $A(z)$ and $B(z)$. Even if both $A(z)$ and $B(z)$ have equiripple magnitude responses, we cannot guarantee that the magnitude response of $H_0(z)$ must be equiripple. To achieve an equiripple magnitude response of $H_0(z)$, we define an error function $E_0(\omega)$ as

$$E_0(\omega) = 1 - |H_1(e^{i\omega/2})|B(e^{i\omega}),$$

and then use Remez exchange algorithm to formulate $E_0(\omega)$ as

$$E_0(\omega_i) = 1 - |H_1(e^{i\omega/2})|B(e^{i\omega}) = ( - 1)^i\delta,$$

where $|H_1(e^{i\omega/2})|$ can be considered to be a weighting function. Hence, $H_0(z)$ will have an equiripple magnitude response in stopband. Similarly, $H_0(z)$ is also required to satisfy a given flatness constraint, that is,

$$\frac{\partial^k|H_0(e^{i\omega})|}{\partial \omega^k} \bigg|_{\omega=0} = 0, \quad k = 0,1,\ldots,2J_2 + 1,$$

where $J_2$ is an integer. Since the flatness degree of $H_0(z)$ is decided by the lower one of flatness between $A(z)$ and $B(z)$, then $J_2 \leq J_1$. From Eq. (28), the flatness constraint of Eq. (43) is equivalent to

$$\hat{B}(e^{i\omega}) = 1,$$

$$\frac{\partial^k\hat{B}(e^{i\omega})}{\partial \omega^k} \bigg|_{\omega=0} = 0, \quad k = 1,2,\ldots,2J_2 + 1.$$ 

Then we can get the linear equations similar to Eq. (34). Therefore, we can formulate the design problem of $B(z)$ in the form of the eigenvalue problem, as shown in Section 3.3. The design algorithm is the same as that in Section 3.4.

4. Design examples

In this section, we present two design examples to demonstrate the effectiveness of the proposed method.

Example 1. We consider design of the minimax filter bank shown in Example 3.2 of [7] with $\omega_p = 0.4\pi$ and $\omega_s = 0.6\pi$ for comparison purposes. In [7], $A(z)$ was a FIR linear phase filter of order 11, and $B(z)$ was the same as $A(z)$. By using the coefficients shown in [7], we computed the magnitude response of $H_0(z)$ and $H_1(z)$, which are shown in Fig. 4 in the dotted line. The stopband attenuation of $H_0(z)$ and $H_1(z)$ were 35.6 and 45.2 dB, respectively. Note that the resulting magnitude responses are different from that shown in Fig. 5 of [7]. We cannot obtain the same result by using the coefficients given in [7]. We use the proposed method to design the IIR linear phase PR filter bank with the same specification. The order of $A(z)$ needs to be $L_1 = 3$ and $L_2 = 2$ only. The magnitude response of $A(z)$ is shown in Fig. 3, and that of $H_1(z)$ is shown in Fig. 4 in the solid line. It is clear that $H_1(z)$ has an equiripple response and the stopband attenuation 45 dB. To control the magnitude error of

![Fig. 3. Magnitude responses of $A(z)$ and $B(z)$ in Example 1.](image)
In this paper, we have proposed a new method for designing two-channel IIR linear phase PR filter banks. Since we have adopted the structurally PR implementation proposed in [7], the PR condition is still satisfied even when all filter coefficients are quantized. From the view point of wavelets, we have shown the design of IIR linear phase PR filter banks with an additional flatness constraint. By using Remez exchange algorithm, we have formulated the design problem in the form of a generalized eigenvalue problem. Therefore, by solving the eigenvalue problem to compute the positive minimum eigenvalue, a set of filter coefficients can be obtained as the corresponding eigenvector. The optimal filter coefficients with an equiripple response can be easily obtained through a few
iterations. The proposed procedure is computationally efficient, and the flatness constraint can be arbitrarily specified.

References